Energy-Efficient Assignment of User Equipment to Cooperative Base Stations

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Abstract—Future radio access networks (RANs) will activate and deactivate base stations (BSs) depending on the activity of users to conserve energy. Reducing the fraction of time a BS is enabled will reduce the energy consumption of the RAN.

Using cooperative transmissions from BSs allows to reach user equipment (UE) that is not in range of any single BS. When some BSs cooperate, other BSs can be deactivated. In this paper, we quantify the reduction in activity probability when BSs can cooperate compared to not cooperating. We assume the BSs are placed on a hexagonal grid and a spatial Poisson process places the UE. We calculate the probability that a BS has to be enabled depending on whether we allow BSs to cooperate or not.

Allowing BSs to cooperate decreases the expected number of enabled BSs per area by up to 11%, depending on the user density. Additionally placing the BSs at the optimal distance for cooperation would reduce activity by additional 39%. We conclude that cooperation can extend the range of BSs and thereby allow other BSs to be disabled and, thus, conserve energy.

I. INTRODUCTION

Future RANs will disable BSs to reduce the energy consumption when the user demand is low. When disabling BSs the two purposes of a RAN still have to be fulfilled: (1) the data rates that UE demands must be satisfied and (2) the RAN must be able to provide a minimal data rate to every position in case a UE wants to declare a demand. We call these purposes covering UE and covering area.

An idle BS draws nearly the same amount of power as a fully loaded BS [1]. To conserve energy, thus, idle BSs should be disabled. When disabling BSs both UE and area still have to be covered. While the area to cover stays the same, the requested data rates change over time. To be able to adapt the provided data rates without losing coverage of an area, signaling and data traffic can be transmitted from different BSs [2]. This allows BSs with long ranges to cover an area and BSs with short ranges to provide the necessary data to the UE. Hence, the BSs with short ranges can be activated and deactivated according to user demands without losing coverage of the area. We will only consider the small BSs in this paper.

One way to reduce the energy consumption of the BSs is to decrease the fraction of time they are enabled. As the UE still has to be covered by enabled BSs, extending the range of the BSs reduces the necessary activity of neighboring BSs. One possible technique for range extension is to use cooperative transmissions from at least two BSs to one UE. In this paper, we analytically quantify the activity probability of BSs and, thus, the energy consumption of RANs. We compare RANs with cooperative BSs to RANs with BSs that cannot cooperate and determine the gain from allowing cooperation.

To determine the activity probability of BSs our related work uses simulations. In contrast to this, we determine it analytically. To be able to do this, we assume the BSs are placed in a hexagonal grid and have a circular radius in which they can provide the requested data rate to the UE. While this is not a realistic model, we only need this model to determine the sizes of overlapping areas and do not consider their actual shape. This allows us to contribute an analytic description of the possible gain in energy efficiency when enabling cooperation in a RAN.

II. RELATED WORK

In one of our older papers [3] we described how cooperative transmission from BSs can reduce the number of BSs necessary to cover an area. In this paper, we describe how many BSs are necessary to cover only the active UE.

Wan et al. [4] provide a PTAS for the NP-hard problem of Minimum Wireless Coverage (MWC). MWC seeks to find the minimum number of disks to cover a set of UE. While this optimally solves a single instance of our problem, we determine the expected number of enabled BSs, when the UE is distributed according to a spatial Poisson process.

Another group of related work uses stochastic geometry [5] to determine outage probabilities, data rates and power consumption under the assumption that both UE and BSs are placed by Poisson processes. Suryaprakash et al. [6], for example, prove that putting BSs into sleep modes is more energy efficient than varying the available bandwidth. We assume BSs to be placed in a hexagonal grid instead of by a Poisson process and determine expected activity probabilities.

Son et al. [7] and Zhou et al. [8] describe heuristics to activate BSs and associate the UE to BSs. Vereecken et al. [9] consider a network of macro- and femtocells. In contrast to our analytical results, they provide results by simulations.

An alternate approach by Fehske and Marsch [10], [11] determines the energy efficiency of cooperative and dense deployments in terms of cost per bit. In contrast to this, we compare the total power consumption to cover the UE.

III. MODEL

Our model consists of an infinite plane with BSs placed in a hexagonal grid, as this is optimal to cover the plane. We assume that a higher layer of macro or signaling BSs exists, which detects the presence and activity of UE. We model only
the micro BSs, which provide the data to the UE and do not cover the area. As micro BSs are usually omnidirectional, we do not consider any directional properties of the antennas. We model only a single moment in time, that is, we do not model the progression of time.

We call the (inter-site) distance between a BS and each of its six neighbors spacing. We assume the average received signal at the UE depends only on the distance from the BS. Hence, the area which a BS can cover is a disk. We define it to have the same radius for all BSs and normalize it to 1.

The plane is populated with UE by a 2-dimensional Poisson process with a density of λ. As we assume the UE to be distributed by a Poisson process, the probability that at least 1 UE is in an area (we will call this “activity in the area”) of size A is the expected activity E(A) = 1 − e−Aλ.

Each UE must be associated to an enabled BS. It can be associated to a single BS or to two cooperating BSs. We consider a cooperative association to be valid, if the sum of both individual signal-to-noise ratios (SNRs) is greater than the SNR of a single connection at maximum range. This abstraction is valid for both maximum ratio combining (MRC) [12] and coherent combining (CC) [13]. As cooperation increases the resource utilization of active BSs the maximum data rate the UE receives will be reduced. We ignore the effects this might have on the duration of the transfers, which will hold for voice and video calls, but not for bulk transfers.

We differentiate between two different forms of cooperation: Limited and unlimited cooperation. Using unlimited cooperation the sum of both signals has to be larger than the threshold given by a single association. Using limited cooperation each of the two individual signals must provide at least half of the required power. This abstraction is valid for both maximum ratio combining (MRC) [12] and coherent combining (CC) [13]. As cooperation increases the resource utilization of active BSs the maximum data rate the UE receives will be reduced. We ignore the effects this might have on the duration of the transfers, which will hold for voice and video calls, but not for bulk transfers.

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1) Areas: In this section we describe the size of the areas which arise using non-cooperative transmission. The largest spacing of non-cooperative BSs that still covers the complete plane is s_{max} = \sqrt{3}. We consider s_{min} = 2/\sqrt{3} as the smallest spacing. While smaller spacings are possible it is necessary to consider new types of overlapping areas to determine the activity probability in these cases.

We denote sizes of the areas by A. Non-cooperative transmission

Using non-cooperative transmission, each UE has to be associated to a single BS. We consider the power consumption of a BS to be independent of load, that means, a disabled BS consumes no power and an enabled BS consumes full power. For reasons of simplicity we assume all BSs have the same power consumption and normalize it to 1. When the power consumption of the BSs is not binary, but linear over load with an activation offset, the gains will be reduced to approximately the fraction of activation offset to full power. Also, we do not consider the additional power needed to do the calculations necessary to for cooperation.

We consider two different metrics of power consumption: (1) The activity probability of BSs P and (2) the expected number of enabled BSs per covered area Q. Note that the average power consumption of a BS is equal to the activity probability under our assumptions. We denote all functions used in this paper with an index “n” for non-cooperative transmission; “s” for sparsely placed BSs with cooperation and “d” for densely placed BSs with cooperation.

In this section, we determine the probability that a BS has to be enabled when all UE has to be covered. The parameters we analyze are the spacing between the BSs and the different transmission technologies. To fairly compare scenarios with different spacing, we consider both activity probability of a single BS as well as the expected number of enabled BSs per area. This allows us to determine the gain of using cooperation in a network which is built for non-cooperative transmission and a network that is built for cooperative use.

To determine the activity probability, we divide the plane into areas that summarize all UE which can be connected to the same set of BSs. From the size of these areas we determine the probability that users are in the area and from that the probability that a BS has to be enabled.

For each scheme we pick only some spacings for which we determine the activity probability. Determining the results for other spacings is merely routine work.

A. Non-cooperative transmission

Using non-cooperative transmission, each UE has to be within 1 unit of distance of an enabled BS. As we are interested in the activity probability of a BS we look at the UE from the perspective of a BS.

1) Areas: In this section we describe the size of the areas which arise using non-cooperative transmission. The largest spacing of non-cooperative BSs that still covers the complete plane is s_{max} = \sqrt{3}. We consider s_{min} = 2/\sqrt{3} as the smallest spacing. While smaller spacings are possible it is necessary to consider new types of overlapping areas to determine the activity probability in these cases.

We denote sizes of the areas by A and add an index for the number of BSs n that can cover it. We additionally label it with e when the area can be covered by exactly n BSs. Figure 1 gives an overview which types of overlapping areas occur and Figure 2 highlights the areas together with their
The areas and the names we assign them for non-cooperative transmissions; shown here for a spacing of $s = 1.4$.

The size of the area that can be covered by 1 (resp. 2) BS is:

$$A_1 = \pi, \quad A_2 = AI(s, 1, 1),$$

where

$$AI(d, r, R) = r^2 \arccos \left( \frac{d^2 + r^2 - R^2}{2dr} \right) + R^2 \arccos \left( \frac{d^2 + R^2 - r^2}{2dR} \right) - \frac{1}{2} \sqrt{(-d + r + R)(d + r - R)(d - r + R)(d + r + R)}$$

is the size of the intersection of two circles with distance $d$ and radii $r$ and $R$ [14]. The overlap of 3 BSs is an equilateral circular triangle [15] with an area:

$$A_3 = \frac{\sqrt{3}}{4} c^2 + 3 \left( \arccos \left( \frac{c}{2} \right) - \frac{c}{4} \sqrt{4 - c^2} \right),$$

where $c$ is the distance between two corners of the equilateral circular triangle, which obeys:

$$c^2 = 3 - \frac{s^2}{2} - s \sqrt{3 - 3s^2/4}.$$

This results in the size of the Areas that can be covered by exactly 2 BSs (resp. 1 BS) to be:

$$A_{2e} = A_2 - 2A_3, \quad A_{1e} = A_1 - 6A_{2e} - 6A_3.$$

To calculate a reference value and to determine the number of BSs per area we additionally need the hexagonal Voronoi area that is closest to a BS:

$$A_h = \sqrt{3}s^2/2.$$

Hence, there are $1/A_h$ BSs per unit area.

2) Activity probability: In this section, we determine (1) the activity probability of a BS and (2) the expected number of enabled BSs per area for non-cooperative BSs.

As a reference value against which to compare the energy-efficient assignment of UE we first determine the activity probability when each UE is assigned to its closest BS. When each UE is assigned to its closest BS a BS has to be enabled if and only if there is activity in $A_h$. Hence the activity probability of a BS under closest assignment is:

$$P_h = E(A_h).$$

For any activity probability $P$ of a BS the expected number of enabled BSs per area is:

$$Q = P/A_h.$$

Determining the exact activity probability of a BS for an energy-efficient assignment is not directly possible as it depends on the activity probability of its neighbors, which in turn depend on the activity probability of their neighbors and so on. As our scenario is symmetric with respect to all BSs, the activity probability $P$ of all BSs is the same. Next, we determine an equation for the activity probability of a BS depending on the activity probability of its neighbors. As all these probabilities are equal, we solve the equation for it.

Consider a BS $B$: it has to cover at least its central area $A_{1e}$. Additionally, consider a neighbor $N$: If $N$ is enabled (probability $P_n$), $B$ can ignore the area which is shared by both $B$ and $N$ (size $A_{2e}$). If $N$ is not enabled (probability $1 - P_n$), the area has to be covered by $B$ if there should be activity. The same argument works for the area $A_3$: When none of the two neighbors that also cover the area are enabled $B$ has to cover it in case of activity. These arguments allow us to calculate the activity probability $P_n$ of BS $B$, based on the activity probability of its neighboring BSs. We do not need to recursively continue as from the symmetry of the scenario all BSs have the same activity probability.

To determine the activity probability, we encode the activity configuration of the neighbors in the binary digits of variable $i$: 0 means disabled and 1 means enabled. We denote the $j$th binary digit of $i$ as $[i/j] = \lfloor i/2^j \rfloor \mod 2$. We iterate over all possible configurations of activity of the neighbors and determine the activity probability of BS $B$ for each configuration through the size of the area that it has to cover.
Probability with limited cooperation can be determined in the same way as without cooperation. It is:

\[ P_n = \sum_{i=0}^{2^d-1} \prod_{j=0}^{5} i[j] P_n + (1 - i[j])(1 - P_n) \]  

(1)

Equation 1 can be simplified to a polynomial of degree 6 in \( P_n \). While this is too complex for a closed form solution, Newton’s method provides us with the means to determine its solution numerically. This solution is the activity probability \( P_n \) of a single BS for the scenario without cooperative transmissions.

**B. Densely placed cooperative BSs**

To compare the activity probability of two association schemes they must have the same spacing. As cooperative transmissions allow a higher spacing we call the placement with the same spacing as the non-cooperative placement dense.

1) **Areas:** Figure 3 provides an overview of the involved areas when the spacing is between \( s_{\text{minD}} = \sqrt[3]{8/3} \) and \( s_{\text{maxD}} = \sqrt{3} \). Figure 4 highlights the relevant areas and provides their names. Note that we only consider limited cooperation for dense cooperative deployments due to the more complex shape of areas with unlimited cooperation.

We call the area that can be covered by exactly two cooperating BSs or one other fixed BS \( A_d \). It can be calculated from the area of intersection of circles:

\[ A_d = \frac{1}{2} \left( A(1, \sqrt{2}, \sqrt{2}) - 2 A(1, \sqrt{2}, 1) + A(1, 1, 1) \right). \]

Hence, the area that is only covered by a single BS has size:

\[ A_{1c} = A_{1c} - 6A_d. \]

2) **Activity probability:** The equation for the activity probability with limited cooperation can be determined in the same way as without cooperation. It is:

\[ P_d = \sum_{i=0}^{2^d-1} \prod_{j=0}^{5} i[j] P_n + (1 - i[j])(1 - P_n) \]

\[ E \left( A_{1c} + A_{2c} \left( \sum_{j=0}^{5} (1 - i[j]) \right) \right) \]

\[ + A_3 \left( \sum_{j=0}^{5} (1 - i[j])(1 - i[j] + 1 \mod 6) \right) \]

\[ + A_4 \left( \sum_{j=0}^{5} 1 - i[j]i[j] + 1 \mod 6 \right) \]  

(2)

**C. Sparsely placed cooperative BSs**

In addition to the dense placement, cooperating BSs can also be placed sparsely.

1) **Areas:** The geometric illustration in Figure 5 shows the areas we consider for cooperative transmissions with a minimum spacing of \( s_{\text{minS}} = 2 \). This lower limit prevents overlap of non-cooperative regions and thus simplifies the calculations. The maximum spacing for limited cooperation is \( s_{\text{maxSL}} = \sqrt{3} \sqrt[3]{2} + 5 / \sqrt{6} \), while the maximum spacing for unlimited cooperation is \( s_{\text{maxSU}} = \sqrt{6} \).

**Fig. 4.** Shapes and names of regions for cooperative BSs with dense placement of spacing \( s = 1.7 \).

**Fig. 5.** Areas in a sparse cooperative placement (with spacing \( s_{\text{minS}} \leq s \leq s_{\text{maxSU}} \) for unlimited cooperation and \( s \leq s_{\text{maxSL}} \) for limited cooperation).

**Fig. 6.** Shapes and names of regions for sparse cooperative placement of BSs with spacing of \( s = 2.2 \).
A lower bound for the size of the area that is covered by exactly two BSs, which is highlighted in Figure 6, is:

\[ A_{se} = \frac{A_h - A_1}{3}. \]

The total area for which a BS has to be enabled is, thus:

\[ A_s = A_1 + 6A_{se}. \]

2) Activity probability: The activity of a BS can be directly determined from the activity in the areas, as no association choices have to be made. The activity probability is:

\[ P_s = E(A_s). \]

V. NUMERICAL RESULTS

In this section, we illustrate the analytical results from the last section with the help of Maxima 5.28.0.

Figure 7 illustrates how cooperative schemes reduce the expected enabled BSs per area. Non-cooperative schemes have a maximum spacing of \( s_{max} = \sqrt{3} \). Cooperative schemes can cover the plane for spacings between \( \sqrt{3} \) and 2, we just did not determine the activity probability in this range. We conclude that it is more energy efficient to operate BSs at a higher spacing, as long as it still covers the area.

Figure 8 shows the expected number of enabled BSs per area relative to the non-cooperative assignment. The dense cooperating scheme is always lower than 1 and, thus, better than the non-cooperative assignment. The sparse cooperative assignment on the other hand has more expected enabled BSs as a single UE can potentially make it necessary to activate 2 BSs to cooperatively provide it with data. When, on the other hand, the user density increases the sparse placement becomes more efficient as it uses fewer BSs.

VI. CONCLUSION

We first determined the sizes of intersections of ranges for a hexagonal placement of BSs in a RAN. From this we calculated the activity probability of the BSs for cooperative and non-cooperative association schemes. Using this result, we determined the expected number of enabled BSs per area.

Our results show that the reduction in power consumption by allowing cooperative transmissions without changing the spacing of the BSs is between 0% and 11% depending on the load. When additionally changing the spacing the power consumption per area can be reduced by additional 39%. We concluded that fewest active BSs are needed if the BSs are placed as far apart as possible, but still cover the area and are able to provide the necessary data rates. Cooperative transmission from the BSs allows to place the BSs further apart than non-cooperative transmission would allow.

Future work will include the limit of data rates the BSs can provide and, thus, determine when the assignment schemes will be data-rate limited instead of coverage limited.

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REFERENCES


